59 [8].—I. J. GOOD, T. N. GOVER & G. J. MITCHELL, *Tables of Smoothed but Almost Exact Distributions of X^2 and of the Likelihood-Ratio Statistic for the Equiprobable Multinomial Distribution*, ms. of 580 computer output sheets (unnumbered) deposited in the UMT file.

Consider a multinomial distribution of t categories for which the physical probability of each category is $1/t$ (the "equiprobable multinomial distribution"). Let a sample of size N be drawn and tlet he frequencies in the t cells be $n_1, n_2, \cdots, n_t$.

Let

$$X^2 = \frac{t}{N} \sum_{i=1}^{t} \left(n_i - \frac{N}{t}\right)^2$$

and

$$\Lambda = 2N \ln t - 2N \ln N + \sum_{i=1}^{t} 2n_i \ln n_i .$$

Both X^2 and Λ have asymptotically chi-squared distributions, but for small samples the chi-squared approximation is sometimes poor. Let $P_1(a)$ and $Q_1(a)$ be the smoothed tail-area probabilities of the exact distributions of X^2 and Λ, that is, the smoothings of $P(X^2 > a)$ and $P(\Lambda > a)$, where the smoothing is performed in the following manner.

Each distribution is a step function, forming a "staircase," and this is first replaced by another "staircase" function obtained by drawing a graph of the negative of the logarithm of the right-hand tail areas. Each step has a horizontal and vertical segment. We bisect all these segments and join adjacent bisecting points by new straight-line segments, each of which thus connects the midpoint of one of the original horizontal segments to the midpoint of one of the adjacent vertical segments. The new segments form, of course, a continuous approximation to the original distribution; however, the derivative is discontinuous at each point of bisection.

At the midpoint between two jumps in the original distribution, the smoothed tail-area probability is equal to the exact unsmoothed one, whereas at a jump it is equal to the geometric mean of the two tail-area probabilities just to the left and just to the right of the jump. At other points the logarithm of the smoothed tail-area probability is determined by linear interpolation, as a consequence of the smoothing procedure described.

The present tables consist of 4D values of the common logarithms of $P_1(a)$ and $Q_1(a)$ for $t = 3(1)6$, $N = 3(1)12$; $t = 6(1)14$, $N = 6(1)2t$; $t = 15(1)18$, $N = 6(1)28$. In both tables, these values appear in columns headed LOGP and the values of a are given in columns headed A. The notations a, P_1, and Q_1 are those used by the authors in a paper [1] for which these tables were computed. These symbols are replaced, respectively, by A, P, and again P in the computer printout.

The tables were computed on the British Science Research Council's Atlas Computer at Chilton, Didcot, Berkshire, England, following many preliminary computations on the CDC 1604 system at the Communications Research Division of the Institute for Defense Analyses, Princeton, New Jersey.

AUTHORS' SUMMARY

1. I. J. GOOD, T. N. GOVER & G. J. MITCHELL, "Exact distribution for X^2 and for the likelihood-ratio statistic for the equiprobable multinomial distribution," *J. Amer. Statist. Assoc.*, v. 65, 1970, pp. 267–283.